CDI-II – Prática – 27/4/21

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Ficha 7: Exercícios 6, 7, 8, 9

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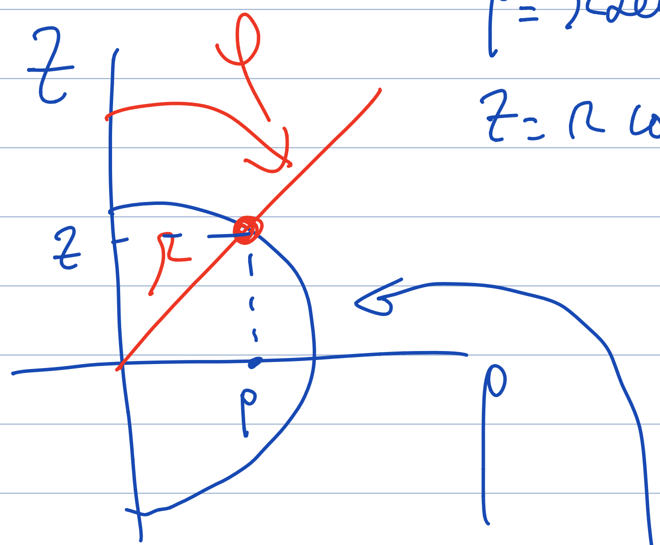
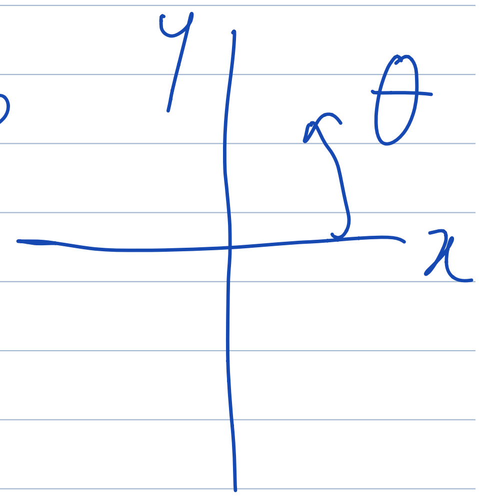
Coordenadas cilíndricas e esféricas

(ρ, θ, z) $\qquad$ (r, θ, φ)

$\rho = r \operatorname{sen} \varphi$

$z = r \cos \varphi$



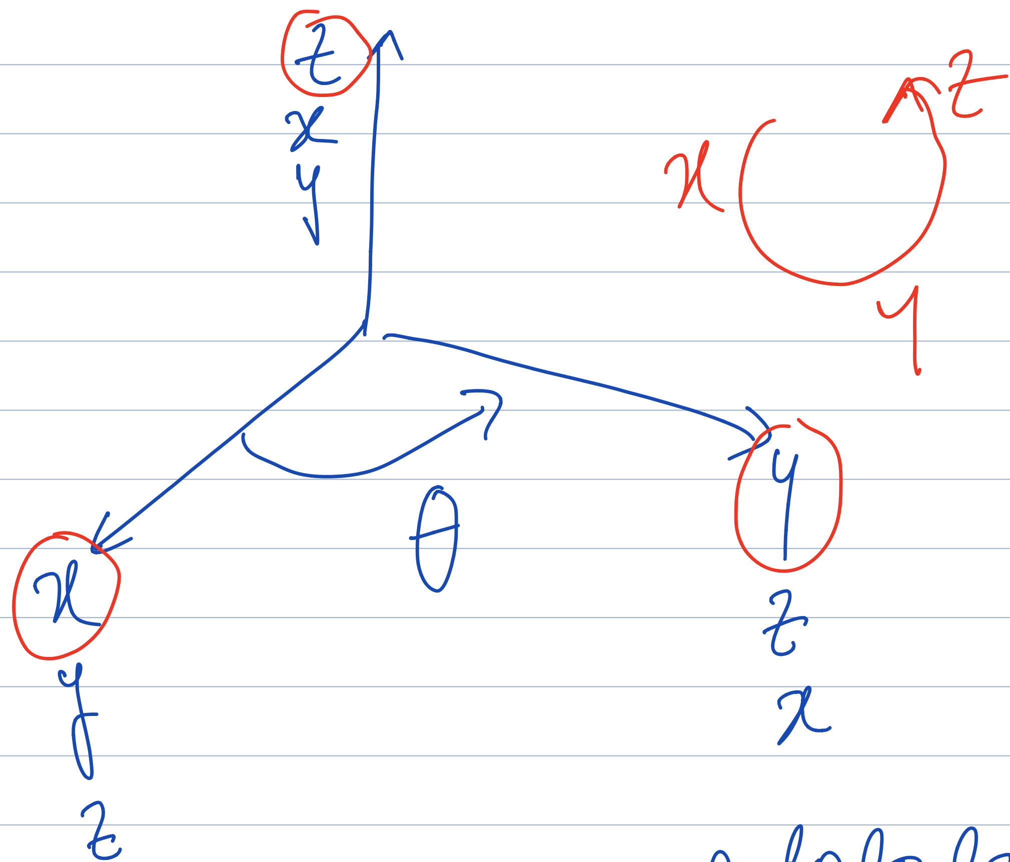


$\theta = \arctan \frac{y}{x}$

$r = \sqrt{x^2+y^2+z^2}$

$$\boxed{r^2 = \rho^2 + z^2}$$

$\rho = \sqrt{x^2+y^2}$



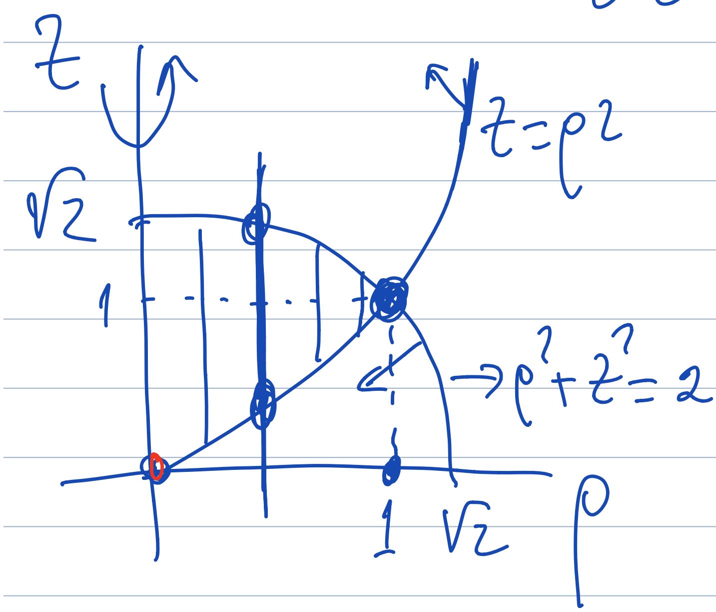
$dx\,dy\,dz$

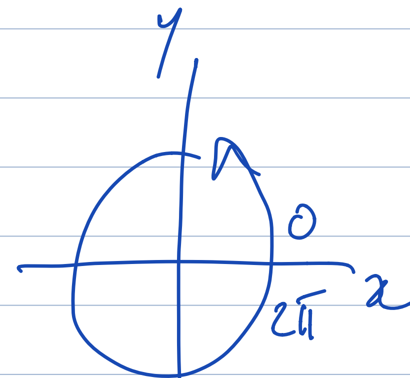
$\rho\, d\rho\, d\theta\, dz$

$r^2 \sin\varphi\, dr\, d\theta\, d\varphi$

5-a) $\rho^2 < z < \sqrt{2-\rho^2}$ ✓

$\rho = \sqrt{x^2+y^2}$ ≡ distância ao eixo Oz.





$$\boxed{0<\theta<2\pi}$$

$$z=\sqrt{2-\rho^2}$$

$$\boxed{\rho^2+z^2=2}$$

$\boxed{\rho\, dz\, d\rho\, d\theta}$ ou

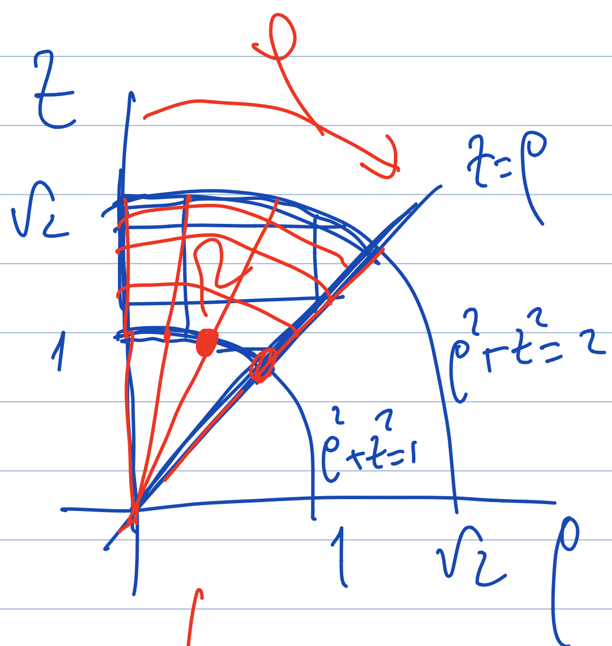
$\rho\, d\theta\, dz\, d\rho$

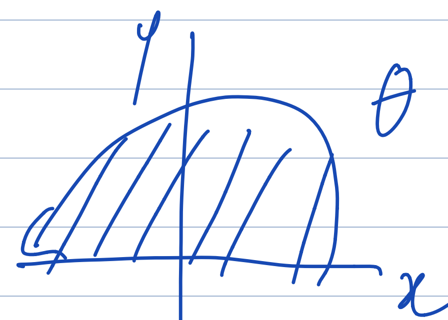
$$\mathrm{Vol}_3(V) = \int_0^{2\pi}\left(\int_0^1\left(\int_{\rho^2}^{\sqrt{2-\rho^2}}\rho\, dz\right)d\rho\right)d\theta \ldots$$

etc

5-b) $z > \rho$, $1 < \rho^2 + z^2 < 2$

$y > 0$





$\boxed{0 < \theta < \pi}$

$0 < \varphi < \frac{\pi}{4}$

$1 < r < \sqrt{2}$

$r^2 \sin\varphi$

$$Vol_3(V) = \int_0^{\pi}\left(\int_0^{\pi/4}\left(\int_1^{\sqrt{2}} r^2 \sin\varphi \, dr\right) d\varphi\right) d\theta$$

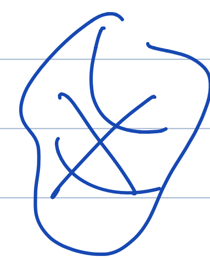
etc...

$\int_X f \qquad X \subset \mathbb{R}^n \; ; \; f: \mathbb{R}^n \to \mathbb{R}$

1) $f \equiv 1 \longrightarrow \int_X f = \mathrm{vol}_n(X)$

2) $f \geq 0$, $f \equiv \sigma$ "sigma"

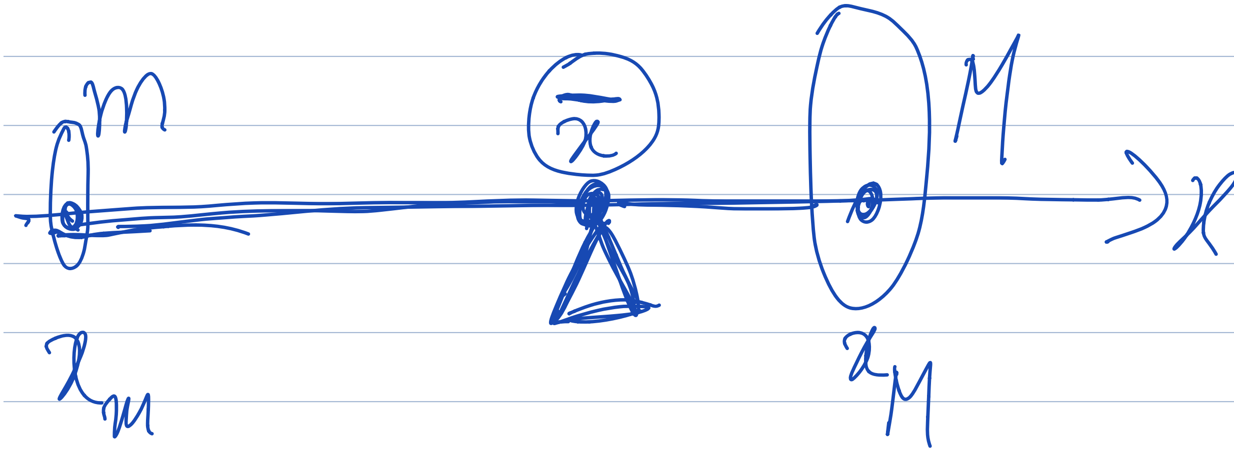
densidade de massa.



$\sigma = \dfrac{M}{V}$ $\longrightarrow$ Massa
$\longrightarrow$ volume

$M = \sigma V = \int_X \sigma$

3 - Centro de massa (centróide)



$$m(\bar{x} - x_m) = M(x_M - \bar{x})$$

$$\bar{x}(m + M) = m x_m + M x_M$$

$$\bar{x} = \frac{m x_m + M x_M}{m + M}$$

$$\bar{x} = \frac{\int_X x\sigma}{\int_X \sigma}$$

$$\bar{y} = \frac{\int_X y\sigma}{\int_X \sigma}$$

$$\bar{z} = \frac{\int_X z\sigma}{\int_X \sigma}$$

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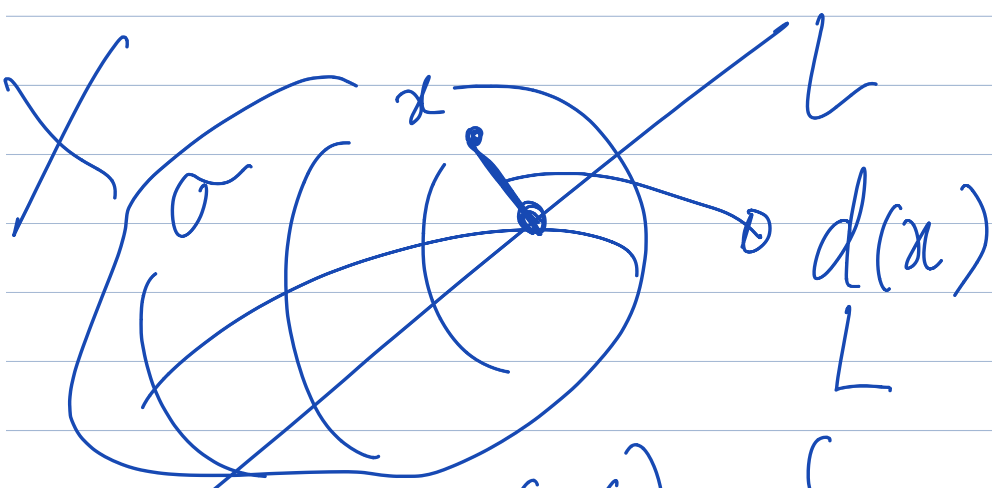
4 — Momento de inércia relativo a uma linha reta L.



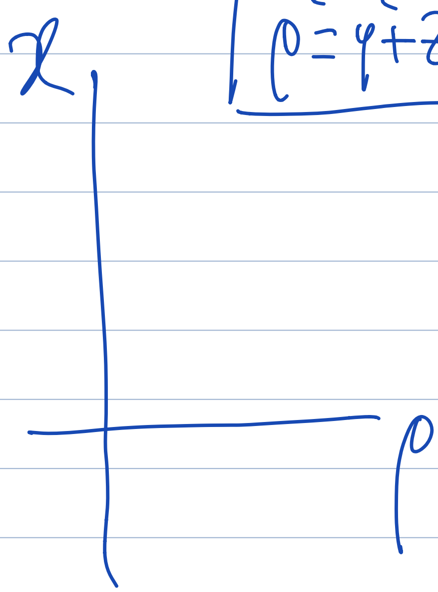
$$I_L(X) = \int_X \sigma \, d_L^2$$

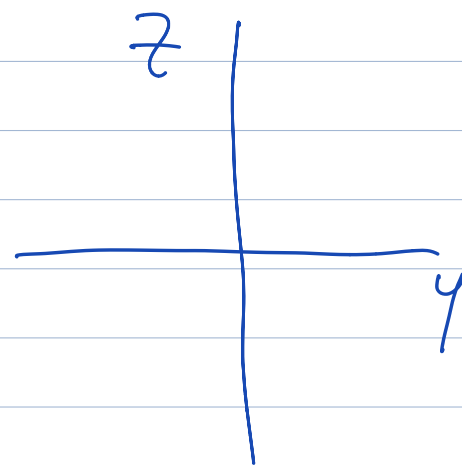
2e $L \equiv Oz$

$$d_L^2(x,y,z) = x^2+y^2.$$

b — $\rho^2 < 1$; $0 < x < \rho^{2/4}$, $y>0$, $z>0$

$\boxed{\rho^2 = y^2+z^2}$





etc.

Relação entre integral e derivada

TFC:

$$\int_a^b f'(t)\,dt = f(b)-f(a)$$

ou

$$\frac{d}{dx}\int_{a(x)}^{b(x)} g(t)\,dt = g(b(x))\,b'(x) - g(a(x))\,a'(x)$$

Leibniz:

$$\frac{d}{dx}\int_T f(x,t)\,dt = \int_T \frac{\partial f}{\partial x}(x,t)\,dt$$

8 — $F(t) = \displaystyle\int_0^1 \underbrace{\operatorname{sen}(tx^2+x^3)}_{f(t,x)}\,dx$

Leibniz

$$F'(t) = \int_0^1 \frac{\partial f}{\partial t}(t,x)\,dx$$

$$F'(t) = \int_0^1 x^2\cos(tx^2+x^3)\,dx$$

$$F'(0) = \frac{1}{3}\int_0^1 3x^2\cos(x^3)\,dx$$

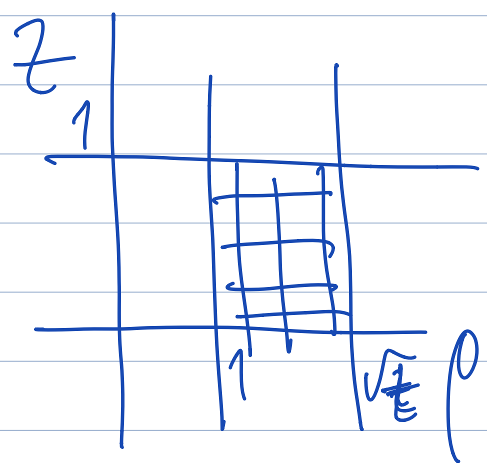
etc ...

9- V_t : $1 < \rho^2 < t$

$0 < z < 1$

$y > 0$ ✓

(ρ, θ, z)



$\boxed{0 < \theta < \pi}$

$$F(t) = \int_1^{\sqrt{t}} \underbrace{\left(\int_0^1 \left(\int_0^{\pi} \rho \frac{e^{t\rho^2}}{\rho^2} d\theta \right) dz \right)}\, d\rho$$

$$F(t) = \pi \int_1^{\sqrt{t}} \frac{e^{t\rho^2}}{\rho}\, d\rho \qquad \sqrt{t} = b(t)$$

TFC + Leibniz.

$$F(t) = I\big(b(t), t\big)$$

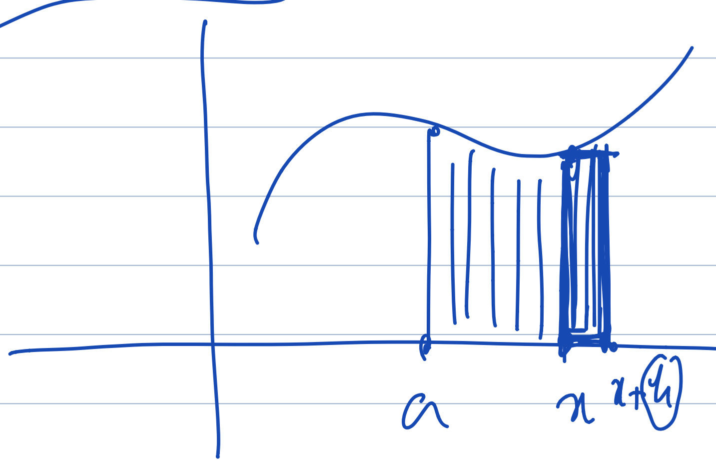
$$F'(t) = \underbrace{\frac{\partial I}{\partial b}\big(b(t),t\big)\, b'(t)}_{\checkmark\ \text{TFC}} + \underbrace{\frac{\partial I}{\partial t}\big(b(t),t\big)}_{\text{Leibniz}}$$

$$F'(t) = \pi \frac{e^{tt}}{2\sqrt{t}\sqrt{t}} + \pi \int_1^{\sqrt{t}} \rho^2 \frac{e^{t\rho^2}}{\rho}\, d\rho$$

$$F'(t) = \pi \frac{e^{t^2}}{2t} + \frac{\pi}{2}\int_1^{\sqrt{t}} 2\rho\, e^{t\rho^2}\, d\rho$$

$e^{t}c\ldots$

$$\frac{d}{dx}\int_a^{x} f(t)\,dt = f(x)$$



d

$$\frac{d}{dx}\int_{a}^{b(x)} f(t)\,dt = f(b(x))\,b'(x)$$

$$I(b(x))$$